Comparison between DOA Estimation Subspace methods for incoherent and coherent signals

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***Abstract*—This paper considers the problem of estimating the direction of arrival (DOA) for the both incoherent and coherent signals from narrowband sources, located in the far field in the case of uniform linear array sensors. Three different methods are analyzed. Specifically, these methods are Music, Root-Music and ESPRIT. The pros and cons of these methods are identified and compared in light of different viewpoints. The performance of the three methods is evaluated, analytically, when possible, and by Matlab simulation. This paper can be a roadmap for beginners in understanding the basic concepts of DOA estimation issues, properties and performance.**

Keywords— Direction of Arrival (DOA); Uniform linear array (ULA); Music; Root-Music; ESPRIT.

I. INTRODUCTION

Research in the array processing is an area of study devoted to processing the signals received from an antenna array and extracting information of interest. It has played an important role in widespread applications like radar, sonar, and wireless communications [1]. DOA estimation of the signals arriving from a particular direction nowadays a quite well established theory and many interesting papers, on this topic, are available in open literature.

During the last decades, many adaptive array processing algorithms have been reported in the literature. Mainly these algorithms, in a general view, exhibit a trade-off between performance and required computational complexity. High resolution DOA estimation is important in many sensor systems. High-resolution frequency estimation is important in different applications as the design and control of robots. Evidently, in such problems, the functional form of the underlying signals can often be assumed to be known e.g., (narrow-band plane waves). The quantities to be estimated are parameters (e.g., frequencies and DOA's of plane waves,) upon which the sensor outputs depend, and these parameters are hence assumed to be stationary [2]. Basically, there have been several approaches to such problems, including the so-called maximum likelihood (ML) method of Capon [3] and Burg's (maximum entropy (ME) method [4].

Even though, these methods are often effective in used and widely employed. Nerveless, these schemes are based on design criteria and assumptions not easily achievable. Therefore, due to this fact, these methods are unpractical and need to be improved. Pisarenko [6] was the first one who tries to exploit the structure of the data model. Schmidt [9] and Bienvenu were the first to perfectly employ the measurement model in the case of sensor arrays of arbitrary form. Alternatively, Schmidt in particular finished this by first deriving a complete geometric solution without the noise, then intelligently expanding the geometric concepts to fulfill acceptable approximate solution in the presence of noise. The resulting algorithm was called MUSIC (Multiple SIgnal Classification) and has been widely considered, among the variety of existing high-resolution algorithms. MUSIC was the most favorable and a leading candidate for further study and actual hardware implementation. However, although the performance advantages of MUSIC are essential, they are accomplished at a significant cost in computation (searching over parameter space) and storage (of array calibration data). The ESPRIT (Estimation of Signal Parameters via Rotational Invariance) algorithm dramatically reduces these computation and storage costs [4].

This paper focuses on how to estimate the DOA of the incoming signal based on aforementioned methods, based on uniform linear array sensors for both coherent and non coherent signals with different environments (numbers of array elements, SNR, number of snapshots). A model for receiving signals is developed for this purpose. Later, the model is subjected to computer simulation to investigate the results and to show the performance.

The rest of the paper is organized as follows: section II reviews the theoretical basics, and the methodology used for this study is outlined in section III. Section IV reports the simulations and Results and lastly in section V focuses on the conclusion and further research.

II. THEORETICAL BASICS

In this section, we consider the theoretical basics of locating n radiating sources by using an array of m passive sensors, as shown in Fig. 1. Generally, the emitted energy from the sources can be electromagnetic wave and the receiving sensors can be any transducers that convert the received energy to electrical signals (antennas).

This problem basically depends on determining how the "energy" is scattered over space with the source positions representing point sin space with concentrations of energy. Thus, hence, it can be named a spatial spectral estimation problem [7]. Meanwhile, this name is also motivated by the fact that there are close ties between the source-location problem and the problem of temporal spectral estimation.

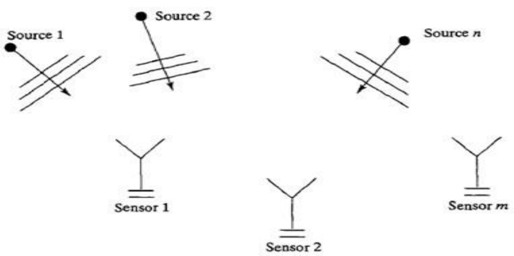

Fig. 1. The set-up of the source-location problem

The advancement of the array model relies on a number of simplifying assumptions. Consequently, some of these assumptions, which have a more general character, are listed below.

- The sources are assumed to be located in the far field of the array.
- Furthermore, we presume that both the sources and the sensors in the array are in the same plane and that the sources are point emitters.
- In addition, it is presumed that the propagation medium is homogeneous (i.e., not dispersive), and therefore the waves coming at the array can be supposed to be planar. Under these instances, the only parameter that describes the source positions is the so-called angle of arrival (AOA), or DOA.
- Moreover, it is presumed that the number of sources n is specified. The selection of n, when it is unknown, is a problem of remarkable importance for many applications, which is often referred as the detection problem.
- Finally, it is assumed that the sensors in the array can be modeled as linear (time-invariant) systems, and thus both their transfer characteristics and their locations are known. In other words, we allege that the array is assumed to be calibrated [2].

A. *Received Signal Model*

Predominately, many of the DOA algorithms rely on the array correlation matrix. Fig. 2 shows D signals arriving from D directions. Clearly, they are received by an array of M elements with M potential weights.

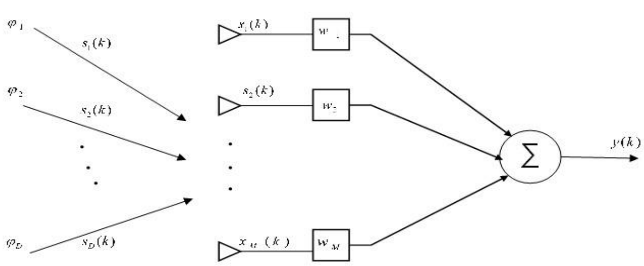

Fig. 2. The set-up of the source-location problem

M-element Array with D Arriving Signals

Each received signal $x_m(k)$ includes additive zero mean Gaussian noise. It should be remarked that, the time is symbolized by the kth time sample. Thus, the array output y can be specified in the following form:

$$y(k) = w^T . x(k) \tag{1}$$

Where he received signal can be expressed as

$$X\,(\mathrm{k}) = [\mathrm{a}_1(\varphi_1)\mathrm{a}_1(\varphi_2)\ldots\mathrm{a}_1(\varphi_D)].\begin{bmatrix} \mathrm{s}_1(\mathrm{k}) \\ \mathrm{s}_2(\mathrm{k}) \\ \vdots \\ \mathrm{s}_D(\mathrm{k}) \end{bmatrix} + \mathrm{n}(\mathrm{k})$$

$$= \mathrm{A}\ .\ \mathrm{s}(\mathrm{k}) + \mathrm{n}(\mathrm{k}) \tag{2}$$

And the weights can show by $\mathrm{w} = [\mathrm{w}_1\mathrm{w}_2\mathrm{w}_3\ldots.\mathrm{w}_M]^T$ = array weights. Furthermore, the rest of parameters can be defined as follows:

$s(k)$ = vector of incident complex monochromatic signals at time k.

$n(k)$ = noise vector at each array element m, zero mean, variance $\sigma_n^{\ 2}$.

$a(\varphi_i)$ = M-element array steering vector for the φ_i direction of arrival.

A is an M×D matrix of steering vectors $a(\varphi_i)$ and is given by:

$$A = [\mathrm{a}_1(\varphi_1) \quad \mathrm{a}_1(\varphi_2) \quad \ldots \quad \mathrm{a}_1(\varphi_D)] \tag{3}$$

Thus, each of the D-complex signals arrive at angles φ_i and is intercepted by the M antenna elements. It is initially assumed that the arriving signals are monochromatic and the number of arriving signals D < M. It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of the incoming signal. Obviously, if the transmitters are moving, the matrix of steering vectors is varying with time and therefore the corresponding arrival angles are also changing.

In order to simplify the notation, let us define the M×M array correlation matrix R_{xx} as

$$\begin{aligned} R_{xx} &= E[x.x^H] = E[(As+n)(s^H A^H + n^H)] \\ &= \mathrm{A\ E[s.s^H]A^H + E[n.n^H]} \\ &= \mathrm{AR_{ss}A^H + R_{nn}} \end{aligned} \tag{4}$$

Where R_{ss} represents the source correlation matrix and $R_{nn} = \sigma_n^{\ 2}\ \mathrm{I} \ = \ M \times M$ represents the noise correlation matrix, whereas I = M×M is the identity matrix.

The array correlation matrix R_{xx} and the source correlation matrix R_{ss} are found by the expected value of the respective absolute values squared (i.e., $R_{xx} = E\left[X.X^{H}\right]$, and $R_{ss} = E\left[S.S^{H}\right]$). If we do not know the exact statistics for the noise and signals, but we can assume that the process is ergodic, then we can calculate the correlation by use of a time-averaged correlation. Hereby, in that case the correlation matrices are defined by

$$R_{xx} \approx \frac{1}{k}\sum_{K=1}^{K} x(K).x^{H}(K) \quad (5)$$

$$R_{ss} \approx \frac{1}{k}\sum_{K=1}^{K} s(K).s^{H}(K) \quad (6)$$

$$R_{nn} \approx \frac{1}{k}\sum_{K=1}^{K} n(K).n^{H}(K) \quad (7)$$

When the signals are uncorrelated the R_{ss} obviously has to be a diagonal matrix because off-diagonal elements have no correlation. When the signals are partly correlated, R_{ss} is non-singular. When the signals are coherent, R_{ss} becomes singular because the rows are linear combinations of each other.

Generally, the goal of DOA estimation techniques is to describe a function that provides a suggestion of the angles of arrival based upon maxima vs. angle. In view of that, this function is conventionally called the pseudospectrum $P(\varphi)$ and the units can be in energy or in watts (or at times energy or watts squared)[3].

B. Uniform Linear Array (ULA)

Consider the array of M identical sensors equally spaced in a line, illustrated in Fig .3. This kind of arrangement of the array is universally referred to as a ULA. Tentatively defines d to be the distance between two adjacent consecutive sensors, and let φ denotes the DOA of the signal enlightening the array, as measured (counter clockwise) with regard to the normal to the line of sensors.

Then, in the consideration of planar wave hypotheses and the assumption that the first sensor in the array is selected as the reference point, we can discover the delay at sensor m, where m = 1, 2, … M. Thereby M can be defined by:

$$\tau_{m} = (m-1)\frac{d \sin \varphi}{c} \quad (8)$$

Where c is the propagation velocity, and it is in the case electromagnetic wave is indicated by the speed of light. Subsequently the phase difference can be computed as:

Linear Array it is assumed that the look direction waveform is uncorrelated with the vector of non-look direction noise, thus the following consideration must take into account which can be expressed as

$$\Delta\varphi_{m} = (m-1)\frac{2\pi}{\lambda} d \sin\varphi = (m-1)\beta d \sin\varphi \quad (9)$$

Where λ is the wavelength and β is the wave number as given by $\beta = \frac{2\pi}{\lambda}$, and d should be smaller than half of the signal wavelength.

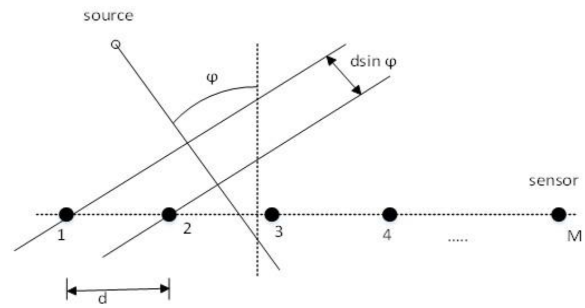

Fig. 3. Uniform Linear Array Structure

Furthermore, we next introduce the so-called array transfer vector, direction vector, and steering vector as:

$$(\varphi) = [1 \quad e^{j(\beta d \sin\varphi)} \quad \ldots \quad e^{j(M-1)(\beta d \sin\varphi)}]^{T} \quad (10)$$

Noteworthy, the steering vector controls the responses of all elements of the array to a source with a single frequency component of unit power. Since the array response is dissimilar in different directions, a steering vector is related with each directional source. The array geometry describes the uniqueness of this association. Thus, for an array of identical elements, each component of this vector has unit magnitude. In addition, the phase of its mth component is equal to the phase difference between signals induced on the mth element and the reference element due to the source associated with the steering vector. The reference element usually is set to have zero phases [5].

The restriction of φ as well as to lie in the interval $[-90^{\circ}, +90^{\circ}]$ is a limitation of ULAs. Moreover, two or more sources at locations symmetric with respect to the array line yield identical sets of delays $\{\tau_{m}\}$ and henceforth cannot be distinguished from one to another. Therefore, in practice these ambiguities of ULAs are removed by using sensors that only pass signals for whose DOAs are limited between $[-90^{\circ}, +90^{\circ}]$.

III. Subspace DOA Estimation Techniques

This section provides an overview of the DOA methods used in our paper. Evidently, the DOA estimation is an essential process to decide the direction of incoming signals and thus to direct the beam of the antenna array towards the estimated direction. Over the years there are enormous varieties of DOA algorithms that have been proposed such as conventional spectral-based, subspace spectral-based and statistical method. The subspace based is considered here and the rest of the methods are not addressed but briefly highlighted. The conventional methods, which estimate DOA through beamforming techniques, are straightforward and require low computation complexity. Nevertheless, these methods have low resolution that leads to the introduction of subspace-based algorithms. Accordingly, high resolution subspace methods, such as MUSIC and ESPRIT, promise preferable accuracy and resolution performance over the conventional methods. However, the performances of these methods greatly depend on the source number estimation and uncorrelated signal environment as well as high computational complexity. In addition, in real applications, when the number of sources is wrongly estimated and the correlated signals existed due to multipath fading, the performance of subspace based methods will deteriorate significantly. Thus, to avoid the problem of source estimation of high computational complexity, Capon algorithm can be applied in DOA estimation, but with the cost of lower resolution compared with subspace-based method.

Nearly, most DOA algorithms, especially the high resolution subspace-method, work skillfully well with the omnidirectional antenna array, but cannot be used directly with a directional antenna array due to three obvious reasons.

- Firstly, the radiation patterns of directional elements are narrow compared with omnidirectional elements.
- Secondly, the mutual couplings between the directional elements are totally considerable and cannot be ignored.
- Thirdly, directional elements have diversified gain on particular signal directions due to the narrow shape of the radiation pattern.

Consequently, all of these characters lead to the difficulty of utilizing the existing DOA algorithms in directional antenna arrays. Thus, as a result, the directional antenna arrays request a DOA algorithm that would fit the characters of directional elements.

Geometrically, the received signal vectors from the received signal vector space whose vector dimension is equal to the number of array elements M. Therefore, the received signal space can be separated into two parts:

The signal subspace and the noise subspace. The signal subspace is the subspace spanned by the columns of $A(\varphi)$, and the subspace orthogonal to the signal subspace is known as the noise subspace. Profitably, the subspace algorithms develop this orthogonality to determine the signals' DOAs.

Generally, there is much helpful information to be observed in the eigen analysis of the array correlation matrix [16]. In the light of M-array elements with D-narrowband signal sources and uncorrelated noise, we can model some assumptions about the characteristics of the correlation matrix. First R_{xx} is an M×M hermitian matrix. A hermitian matrix is equal to its complex conjugate transpose. Basically, the antenna array correlation matrix has M eigenvalues $(g_1, g_2, \ldots, g_M)$ along with M associated eigenvectors $E = [e_1 \ e_2 \ \ \ldots \ \ e_M]$. Thus, if the eigenvalues are sorted from smallest to largest, we can separate the matrix E into two subspaces such that $E = \left[E_N \ \ E_S\right]$. The first subspace E_N is called the noise subspace and it is composed of M−D eigenvectors associated with the noise. On the other hand, the second subspace E_S is called the signal subspace and it is composed of D eigenvectors associated with the arriving signals. We should point out that, the noise subspace is an M× (M−D) matrix whereas the signal subspace is an M×D matrix. Next we will investigate the methods that chosen in this paper.

A. *The MUSIC Algorithm Array*

MUSIC is an abbreviation which stands for Multiple Signal Classification. Mainly, this approach is a popular high resolution eigen structure method which was first posed by Schmidt [9]. MUSIC assures to give unbiased estimates of the number of signals, the angles of arrival, and the strengths of the waveforms. MUSIC formulates the assumption that the noise in each channel is uncorrelated, therefore making the noise correlation matrix diagonal. In addition, the incident signals may be somewhat correlated, creating a non-diagonal signal correlation matrix. Nevertheless, under high signal correlation the conventional MUSIC algorithm breaks down and other methods must be realized to rectify this weakness.

One has to know in advance the number of incoming signals or he should search the eigenvalues to decide the number of arriving signals. If the number of signals is D, the number of signal eigenvalues and eigenvectors is D, and the number of noise eigenvalues and eigenvectors is M−D (M is the number of antenna array elements). Meanwhile, because MUSIC takes advantage of the noise eigenvector subspace, it is sometimes referred to as a subspace method. As for DOA estimation, we compute the array correlation matrix assuming uncorrelated noise with equal variances.

$$R_{xx} = AR_{ss}A^H + \sigma^2_n I \tag{11}$$

Next we find the eigenvalues and eigenvectors for R_{xx}. Then we produce D eigenvectors associated with the signals and M−D eigenvectors associated with the noise. Also, we choose the eigenvectors associated with the smallest eigenvalues. In a situation of uncorrelated signals, the smallest eigenvalues are equal to the variance of the noise. Thus, we can then construct the M× (M−D) dimensional subspace spanned by the noise eigenvectors.

As pointed out earlier the noise subspace eigenvectors are orthogonal to the array steering vectors at the angles of arrival. Based on the orthogonality condition, therefore one can show that the Euclidean distance $d = a(\varphi)^H E_N E_N{}^H a(\varphi) = 0$ for each/every arrival angle. By placing this distant expression will guarantee creates sharp peaks at the angles of arrival. The MUSIC pseudo spectrum is now given as:

$$P_{MU}(\varphi) = \frac{1}{a^H(\varphi) E_N E_N{}^H a(\varphi)} \tag{12}$$

The significant problem of MUSIC is that the accuracy is limited by the discretization at which the MUSIC function

$P_{MU}(\varphi)$ is evaluated. More importantly, it requires either human interaction to decide on the largest M peaks or a comprehensive search algorithm to determine these peaks. This is an extremely computationally intensive and also processing array calibration is critical. Therefore, MUSIC by itself is not very practical; we require a methodology that results directly in numeric values for the estimated directions. This is where the Root MUSIC comes in, which provide better performance than music especially in low SNR situations, and it only worries about the phase of the roots.

B. Root-MUSIC AOA Estimation

Root-MUSIC implies that the MUSIC algorithm is reduced to finding the roots of a polynomial as opposed to only plotting the pseudospectrum or searching for peaks in the pseudospectrum.

MUSIC algorithm is simplified for the case where the antenna is a ULA. Recalling that the MUSIC pseudospectrum is given by (12), thus one can facilitate the denominator expression by defining the matrix $C=E_N$ which is hermitian. Accordingly, this leads to the root-MUSIC expression which can be recast as

$$P_{RMU}(\varphi) = \frac{1}{a^H(\varphi)\, C\, a(\varphi)} \tag{12}$$

We consider a scenario with the ULA, the mth element of the array steering vector is given by $a_m(\varphi) = e^{j\beta d(m-1)\sin\varphi}$ where $m = 1, 2\ldots M$. The result of the denominator argument can be written as

$$\begin{aligned} a^H(\varphi) C a(\varphi) &= \sum_{m=1}^{M} \sum_{n=1}^{N} e^{-j\beta d(m-1)\sin\varphi} C_{mn} e^{j\beta d(n-1)\sin\varphi} \\ &= \sum_{l=-M+1}^{M-1} C_l e^{j\beta d l \sin\varphi} \end{aligned} \tag{13}$$

Where C_1 is the total sum of the diagonal elements of C along the lth diagonal such that $c_l = \sum_{m-n=l} c_{mn}$ .It should be noted that the matrix C has off-diagonal sums such that $c_0 > |c_l|$ for $l \neq 0$. Thus the sum of off-diagonal elements is permanently lower than the sum of the main diagonal elements. In addition, For a 6 × 6 matrix we have 11 diagonals ranging from diagonal numbers $l = -5, -4, \ldots, 0, \ldots, 4, 5$. The lower left diagonal is represented by $l = -5$ whereas the upper right diagonal is represented by $l = 5$. The C_1 coefficients are calculated by $c_{-5} = c_{61}, c_{-4} = c_{51} + c_{62}, c_{-3} = c_{41} + c_{52} + c_{63}$, and so on.

We can simplify the aforementioned equations to be in the form of a polynomial whose coefficients are C1, thus yielding

$$D(Z) = \sum_{l=-M+1}^{M-1} c_l z^l \tag{14}$$

Where $Z = e^{-j\beta d \sin\varphi}$

It is worth remarking that, the roots of D (z) that lie closest to the unit circle correspond to the poles of the MUSIC pseudospectrum [8]. Thus, this technique is called the root-MUSIC. The polynomial is of order 2(M−1) and therefore has roots of $z_1, z_2, \ldots, z_{2(m-1)}$. Each root can be complex and using polar notation can be written as

$$z_i = |z_i|\, e^{j\arg(z_i)} \qquad i = 1, 2, \ldots, 2(M-1) \tag{15}$$

Where arg (z_i) represents the phase angle of z_i.

Exact zeros in D(z) exist when the root magnitudes |zi| = 1. Then the calculation of the AOA achieved by comparing $e^{j\arg(z_i)}$ to $e^{-j\beta d \sin\varphi}$ and finally the AOA can be given by

$$\varphi(i) = -\sin^{-1}\left(\frac{1}{\beta} \arg(z_i)\right) \tag{16}$$

C. The ESPRIT AOA Estimation

The target of the ESPRIT technique is to develop the rotational invariance in the signal subspace, which is formed by two arrays with a translational invariance structure. ESPRIT essentially assumes narrowband signals so that one knows the translational phase relationships between the multiple arrays to be used. As with MUSIC and ESPRIT assume that there are D < M narrow-band sources centered at the center frequency f_0. In general, these signal sources are assumed to be of a sufficient range, so that the incident propagating field is approximately planar [9]. The sources can be either random or deterministic and the noise is assumed to be random with zero mean.

ESPRIT supposes multiple identical arrays called doublets. These can be divided the arrays or can be composed of sub-arrays of one larger array. It is important to remark that these arrays are displaced translational but not rotationally. An example is shown in Fig. 4 where a four element linear array is composed of two identical three-element sub-arrays or two doubles. These two sub-arrays are translational displaced by the distance d. Let us depict these arrays as Array 1 and Array 2.

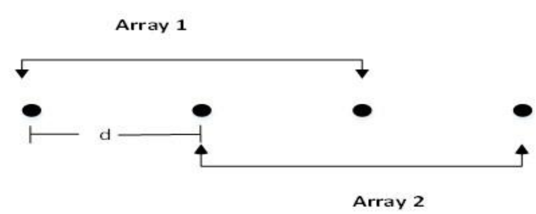

Fig. 4. Doublet Composed of Two Identical Displaced Arrays

The signals induced in each of the arrays are given by

$$\begin{aligned} x_1(k) &= [a_1(\varphi_1) a_1(\varphi_2) \ldots a_1(\varphi_D)] . \begin{bmatrix} s1(k) \\ s2(k) \\ \vdots \\ sD(k) \end{bmatrix} + n_1(k) \\ &= A_1 . s(k) + n_1(k) \end{aligned} \tag{17}$$

And
$$\begin{aligned} x_2(k) &= A_2.\, s(k) + n_2(k) \\ &= A_1.\Phi.(k) + n_2(k) \end{aligned} \tag{18}$$

Where $\Phi = diag\{e^{j\beta d\sin\varphi_1}, e^{j\beta d\sin\varphi_2}, ..., e^{j\beta d\sin\varphi_D}\}$ which is a DxD diagonal unitary matrix with phase shift between the doublets for each DOA. Whereas A_i expresses the Vandermonde matrix of steering vectors for sub-arrays i=1, 2.

The complete received signal, in view of the contributions of both sub-arrays, is given as:

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} A_1 \\ A_1.\Phi \end{bmatrix}.(k) + \begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix} \tag{19}$$

Then, We can now calculate the correlation matrix for either the complete array or for the two sub-arrays, to this end, correlation matrices can be decomposed into two subspaces $E_1$and E_2 .Then, the subspaces of eigenvectors are related by a unique non-singular transformation matrix Ψ such that $E_1\Psi = E_2$ and so must also exist a unique non-singular transformation matrix T such that $E_1 = AT$ and analogously $E_2 = A\Phi T$.

Finally, after some rearrangements in the previous consecrations and assuming that A is of full-rank, we can derive the relationship bellow

$$T\Psi T^{-1} = \Phi \tag{20}$$

Ψ is a rotation operator that maps the signal subspace E_1into the signal subspace E_2 .Thus; it is profitably to leave the problem of estimating the subspace rotation operator Ψ and consequently finding the eigenvalues of Ψ

If we are constrained to a finite number of measurements and we also assume that the subspaces E_1 and E_2 are equally noisy, we can estimate the rotation operator Ψ using the total least squares (TLS) criterion. This procedure is summarized as follows:

1) Estimate the array correlation matrices from the data samples.

2) Knowing the array correlation matrices for both subarrays, one can estimate the total number of sources by the number of large eigenvalues in either $R_{11}$and R_{22} .

3) Calculate the signal subspaces $R_{11}$and R_{22} based upon the signal eigenvectors of the For $R_{11}$and R_{22} ULA, one can instead construct the signal subspaces from the entire array signal subspace E_s. Es is an M × D matrix composed of the signal eigenvectors. E_1 can be constructed by selecting the first M/2 + 1 rows ((M + 1)/2 + 1 for odd arrays) of $R_{11}$and R_{22} .E_2 can be constructed by selecting the last M/2+1 rows ((M+ 1)/2 + 1 for odd arrays) of E_s.

4) Next structure a 2D × 2D matrix using the signal subspaces such that.

$$C = \begin{bmatrix} E_1{}^H \\ E_2{}^H \end{bmatrix} [E_1 E_2] = E_c \Lambda E_c{}^H \tag{21}$$

where the matrix E_C is from the eigenvalue decomposition (EVD) of C such that $g_1 \geq g_2 \geq ...\ g_{2D}$ and $\Lambda = diag\{g_1, g_2, ...,\ g_{2D}\}$ which splitting E_C into four D × D submatrices such that:

$$E_c = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \tag{21}$$

5) Estimate the rotation operator Ψ by and then Calculate the eigenvalues of Ψ .

6) Lastly estimate the AOA given that $g_i = |g_i| e^{j\arg\arg(g_i)}$ by

$$\varphi(i) = \sin^{-1}\left(\frac{\arg(g_i)}{\beta d}\right) \tag{22}$$

ESPRIT is more robust with respect to array imperfections and has better resolution. Moreover, ESPRIT reduces the computations and storage costs, and also no calibration needed. Furthermore, ESPRIT deals with many conditions that are not easy to achieve. On the negative side, it needs doublets, and must calculate the total least-squares (TLS) or least-squares (LS).

D. Spatial Smoothing

There is a great effort has been spent in developing high resolution techniques for estimating the DOA of multiple signals by multiple sensors. These methods in common, use specific eigenstructure properties of the sensor array output covariance matrix and are known to yield high resolution even when the signal sources are partially correlated. However, when some of the signals are completely correlated (coherent), as happens, for example, in multipath propagation, these techniques encounter serious difficult ties. Many research works have been proposed take care of this situation, particularly ,their solution is based on a preprocessing scheme that partitions the total array of sensors into sub-arrays and then produces the average of the sub-array output covariance matrices. By this way it is possible to estimate all directions of arrival irrespective of their degree of correlation [12].

Spatial smoothing is a solution to the coherent case problem for the ULA case. Suppose a ULA with M sensors are divided into overlapping sub-arrays with L sensors. Sensors {1… L} form the first sub-array sensors, sensors {2… L+1} form the second sub-array as can be seen in Fig. 5.

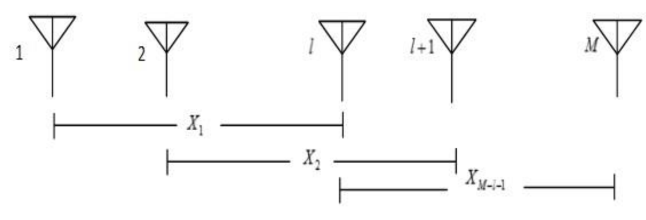

Fig. 5. Spatial Smoothing Scheme

Accordingly, the spatial smoothing covariance matrix is defined by calculating the Average of the individual sub-array covariance matrices.

$$\hat{R} = \frac{1}{b}\sum_{K=1}^{b} R_K \quad (22)$$

Where $b = M - L + 1$ is the number of sub-arrays.

Clearly, the spatial smoothing method reduces the effective aperture (capture area), which is the frontal from which a receiving antenna extracts energy from passing electromagnetic waves. Furthermore, there is an improved spatial smoothing scheme-called the forward/backward spatial smoothing scheme [13], which reduces the number of elements needed for estimating the DOA. The enhancements are carried out by instead of using only the forward sub-array, the scheme makes use of the complex Conjugated backward sub-arrays of the original array to achieve better performance.

IV. Performance Analysis

The performance analysis is conducted by simulations. To this end, MUSIC, ROOTMUSIC and ESPRIT techniques for DOA estimations are simulated using both coherent and uncorrelated signals.

At first, in the case the uncorrelated signal, we consider a system with 10 typical elements in the ULA (M=10), with sensors separated by a half-wavelength d=0.5λ and SNR=10dB, with noise variance = 0.The number of samples to be generated (snapshots) taken to be N =300. Lastly, we assume there is two received uncorrelated, power-equal signals at the angle = [20˚ 40˚].

The response of MUSIC is shown in Fig. 6, whereas Table (1) shows the estimated angle by Root MUSIC and ESPRIT. Table (1) also shows the comparisons of the performance efficiency based Elapsed time.

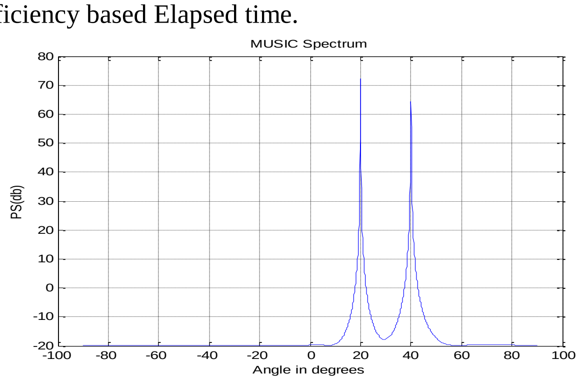

Fig. 6. The response of MUSIC

TABLE I. THE RESPONSE OF ROOT MUSIC AND ESPRIT

Table Column Head		
Characteristics	***Root MUSIC***	***ESPRI***
Angles estimated in degree	19.9941 39.9713	19.9989 39.9918
Elapsed time in seconds	0.014534	0.005430

From Fig. 6, we can see clearly that MUSIC is sharpest in the peak and the bandwidth is very small, that indicates MUSIC has better resolution and accuracy.

From the table I above, it is clear that the ESPRIT algorithm slightly performs Root MUSIC algorithm in the AOA but with remarkable enhancements in reducing the running time.

Secondly, we will compare the MUSIC method in light of different viewpoints as indicated earlier. The comparison is carried out by changing the number of antenna array elements, SNR and the snapshots with the same assumption in the simulation above. Fig. 7 highlights the performance with different numbers of Elements while Fig. 8 shows the performance with different number of snapshots and Fig. 9 points out the performance when different SNR is conducted.

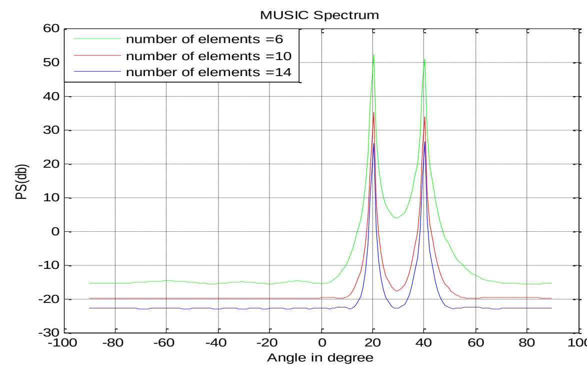

Fig. 7. MUSIC with different number of Elements

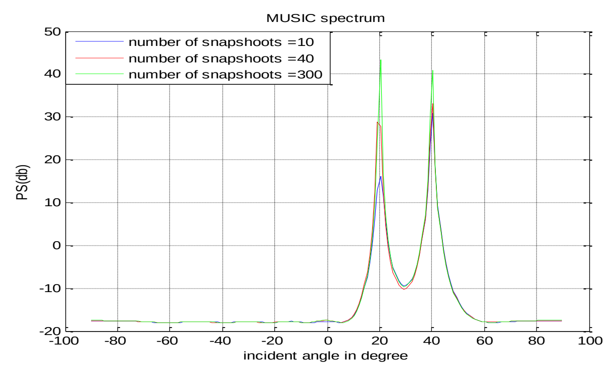

Fig. 8. MUSIC with different number of snapshots

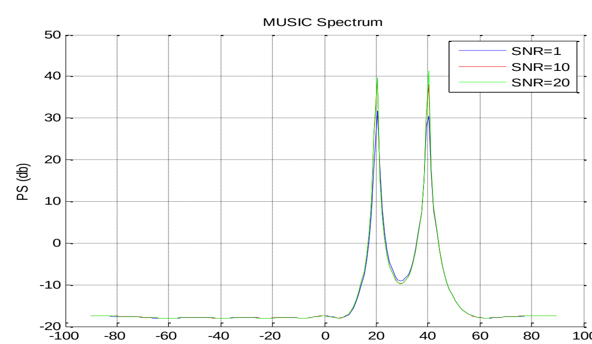

Fig. 9. MUSIC with different SNR

For Fig. 7, the SNR=10, the number of noise variance = 0.1 and snapshots k=300. The results indicate that, as the array size increases from 6 to 14, peaks in the spectrum become sharper. Henceforth, resolution capability increases and MUSIC become more robust.

In Fig. 8, the snapshots increase from 10 to 300.Therefore, resolution capability increases, and the two signals can be clearly identified. Similarly, in Fig. 9 as the SNR decreases from 20 to 1, peaks in spectrum start to disappear and other peaks start to rise and hence resolution capability for closely spaced signal decreases.

From the above comparisons we can observe that an unavoidable relationship between computational complexity and higher performance. Furthermore, the results show that the MUSIC algorithm depends on antenna elements more than the snapshots and SNR because there is no remarkable enhancement with increasing the snapshots and the SNR as comparable to the antenna elements. Moreover, all the algorithms in low SNR, a small number of snapshots and a small number of antenna elements may lead to inaccurate or wrong estimates of DOAs. However, some schemes perform better than others in making this tradeoff.

Thirdly, we then compare the root mean square errors (RMSE) for Root MUSIC and ESPRIT in different number of antenna array elements. Fig. 10 shows the comparison.

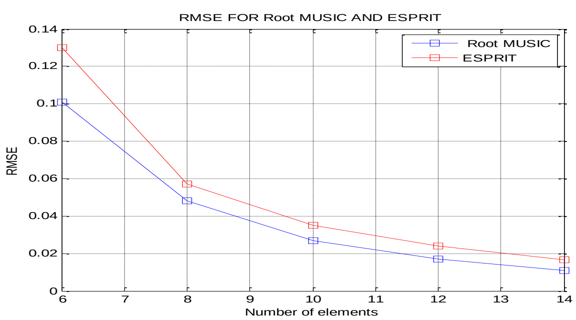

Fig. 10. Root MUSIC and ESPRIT with different number of elements

From Fig. 10, it is obvious to see that the rate of change in the errors for Root MUSIC and ESPRIT approximately constant. Fig. 10 also verifies that a poorer estimate generally results when using a small number of elements.

Finally, we will demonstrate the effectiveness and the advantages of the spatial smoothing. Simulation results are presented to illustrate the performance of the spatial smoothing scheme to solve the coherent signal situations.

The array of 10 elements is divided into 6 sub-arrays, each with a length of 5 elements, and the others parameters kept as above. Fig. 11 shows that the coherent signals appear like one signal when received.

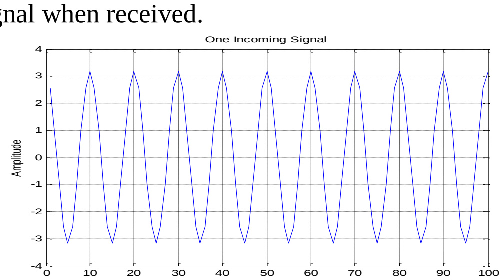

Fig. 11. The coherent signals

From the Fig. 11 above, it is confirmed that the coherent signals give the impression like one signal, but actually it is two signals with the same frequency and same initial phase.

Fig. 12 plots the MUCIC methods with the spatial smoothing technique. Therefore, by combining MUCIC with spatial smoothing the dilemma of coherent signals DAO estimation can be easily solved. Table II shows the Root MUSIC and ESPRIT spatial smoothing response. Accordingly, we can indicate that both the methods are accurate and they approximately give a same DOA estimation in the coherent signal environments.

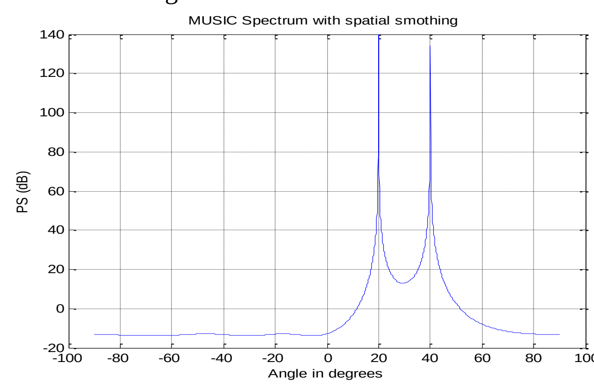

Fig. 12. MUCIC with spatial smoothing

TABLE II. ROOT MUSIC AND ESPRIT SPATIAL SMOOTHING

Table Column Head		
Characteristics	***Root MUSIC***	***ESPRI***
Angles estimated in degree	19.9993 39.9987	19.9988 39.9964
Elapsed time in seconds	0.012345	0.004603

V. CONCLUSION

This paper considers a comparison between DOA estimation subspace methods in both incoherent and coherent signals. Nowadays, MUSIC, Root MUSIC and ESPRIT have developed rapidly and are widely applied for DOA estimation and many others useful applications. Therefore, these methods require further consideration and attention whilst considering reliability. Moreover, the improvement of these algorithms that will be of practical use depends both on the use of accurate designs and the use of designs that are sufficiently simple to permit tractable mathematical analysis.

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